

Short Description of the Scientific Work
(presented in International Journals and Conferences)
of Dr. Nicholas P. Karampetakis

My scientific work is focused on developing algebraic-polynomial methods for the analysis and synthesis of linear, time-invariant, multivariable automatic control systems. Polynomial and polynomial matrix methods are modern industrial design techniques for complex continuous and discrete-time, multivariable control systems, digital filters, signals and processes based on manipulations with polynomials, polynomial matrices, and other similar mathematical objects. Invented, developed to leading world level and applied in Europe, the methods are considered typically European. The models studied are described by linear differential equations with coefficients tables having the form:

$$A(\rho)\beta(t) = B(\rho)u(t) \quad (1a)$$

$$y(t) = C(\rho)\beta(t) + D(\rho)u(t) \quad (1b)$$

where $\rho := d/dt$ denotes the differential operator, $A(\rho) \in \mathbb{R}[\rho]^{r \times r}$ with $\det[A(\rho)] \neq 0$, $B(\rho) \in \mathbb{R}[\rho]^{r \times m}$, $C(\rho) \in \mathbb{R}[\rho]^{p \times r}$, $D(\rho) \in \mathbb{R}[\rho]^{p \times m}$, $u(t) : [0-, +\infty) \rightarrow \mathbb{R}^m$ is the input of the system, $\beta(t) : [0-, +\infty) \rightarrow \mathbb{R}^r$ is the pseudostate of the system and $y(t) : [0-, +\infty) \rightarrow \mathbb{R}^p$ is the output of the system. The above descriptions are also used in discrete time systems with the only difference that in place of the differential operator (ρ) we use the shift operator (σ) i.e. $\sigma x(t) = x(t+1)$ and the domain $[0-, +\infty)$ is replaced by the ring of integers \mathbb{Z} . The frequency representation of (1) is given through the following figure :



Picture 1. Frequency domain representation of the system (1).

where $G(s) = C(s)A(s)^{-1}B(s) + D(s)$ is the transfer function of (1) and $Y(s)$, $U(s)$ are the Laplace transforms of the output and input vector function respectively.

The description presented in (1) is known as a Polynomial Matrix Description (PMD) and is used in the description of large scale systems; examples we have also in power systems [Sto1], [Man1] and the interconnected systems (interconnected systems) [Ros3]. Recent applications we have in robotics and neutral delay systems [Spo1], in aircraft dynamics with imposed algebraic relations for steady-state trim [Ste1], neurological events [Zee1], [DeClaRin1] and the catastrophic behaviour [SasDes1].

The main objectives of my research are focused on:

1. Study of the mathematical structure of linear, time invariant, multivariable automatic control systems.

a) Analytical formulae for the solution of continuous and discrete time systems of the form (1) i.e. analytical formulae of the pseudostate vector $\beta(t)$ and the output vector $y(t)$ under known input $u(t)$ and initial conditions of the pseudostate $\beta(t)$ ([A15], [A19], [A23], [A26], [A41], [B15], [B20], [B21], [B28], [B31], [B50]).

b) Study of the smooth and impulsive solution space (forward and backward solution space in discrete time case) of the homogeneous system of algebraic and differential equations of the form $A(\rho)\beta(t) = 0$ ($A(\sigma)\beta(k) = 0$ in the discrete time case) where the polynomial matrix $A(\rho)$ is not necessarily square and nonsingular ([A18], [A20], [A23], [A30], [B5], [B8], [B10], [B18], [B34], [B35], [B43], [B44]).

c) Study of structural properties of discrete and continuous time systems of the form (1) such as controllability, observability (observability), minimality, structure of square inverse systems e.t.c. ([A7], [A14], [A15], [A17], [A20], [A41], [A48], [B9], [B12], [B17], [B18], [B26], [B30], [B50], [B70]).

d) Study of transformations between continuous or discrete time systems of the form (1), as well as between AutoRegressive representation, that keeps invariant certain properties of the system related with its smooth and impulsive behavior (forward and backward behavior for discrete time systems) ([A1], [A2], [A3], [A5], [A6], [A7], [A8], [A9], [A14], [A17], [A22], [A27], [A29], [A32], [A37], [A43], [B1]-[B4], [B6], [B9], [B11], [B13], [B14], [B16], [B26], [B32], [B37], [B42], [B45], [B47], [B52], [B56], [B72]).

e) Study of synthesis problems of linear, time invariant, multivariable systems, as for example the pole placement problem in $\mathbb{C} \cup \{\infty\}$ which has numerous applications i.e. deadbeat controller, stabilizing controller e.t.c. ([A2], [A10], [A12], [A13], [A14], [A16], [A42], [B2], [B14], [B19], [B22], [B23], [B33], [B38]).

f) Discretization of generalized state space systems ([A28], [A50], [B65], [B71]).

g) Minimal realization of nonproper transfer function matrices ([A40], [B12], [B61]).

i) Construction of systems of algebraic and differential equations (algebraic and difference equations for discrete time systems) with prescribed smooth and impulsive solution space (forward and backward behavior) ([A47], [B67]).

j) Analysis and synthesis of 2-D systems ([A4], [A13], [A31], [A39], [A43], [A45], [B7], [B24], [B49], [B51], [B53], [B54], [B57], [B58], [B59], [B60]).

2. Development of numerical and symbolical algorithms for the analysis and synthesis of Automatic Control Systems.

The methodology that I and my colleagues are used for software development, is known in the literature as “*the algebraic and polynomial matrix approach*”. This methodology is based on the representation of linear dynamical systems by what is known in the literature as Polynomial Matrix Descriptions (PMDs) (see (1)). This methodology started to be developed 30 years ago in the University of Manchester Institute of Science and Technology (UMIST) by H.H. Rosenbrock [Ros1], [Ros2]. It was further developed in the U.S. initially by Wolovich [Wol1] at Brown University and later mainly by Kailath [Kail1] at Stanford, Callier and Desoer [KalDes1] at Berkley, Vidyasagar [Vid1] at MIT and Antsaklis [Ants1] at the University of Notre Dame. In Europe important contributions to this theory were made initially by Kucera [Kuc1] and Sebek in the Institute of Control and Automation of the Czechoslovak Academy of Science in Prague, Blomberg and Ylinen [BIY11] in Finland and by Vardulakis and Karcanias initially at Cambridge University in the U.K. and subsequently by Karcanias and Vardulakis [3] and their colleagues and students respectively at The City University, London, U.K. and the Aristotle University of Thessaloniki, Thessaloniki, Greece.

Although the use of polynomials and polynomial matrices is the natural framework for the description of linear multivariable systems, the lack of reliable and efficient algorithms for their manipulation is a serious disadvantage. During the last decade there was a tendency of change to this state of affairs

which was mainly due to the appearance of a new generation of algorithms and techniques based, amongst others, on Fast Fourier Transforms (FFT) a tool which has proved to be very powerful and successful in signal processing. Therefore my research was focused on using existing methods and develop new fast and reliable algorithms for the support of the algebraic and polynomial approach used in Analysis and Synthesis of Control Systems. A brief description of the work that I have done in this area is to develop algorithms (numerical and symbolic) for :

- a) the inversion of one variable or multivariate polynomial matrices, by using numerical analysis techniques as like as DFT transforms, interpolation methods e.t.c., with applications to the calculation of transfer function matrices ([A4], [A11], [A31], [A44], [A46], [B7]).
- b) the computation of the generalized inverse of a polynomial/rational matrix and the Drazin inverse of a square polynomial matrix with applications to the numerical solution of polynomial matrix diophantine equations, numerical solutions of systems of algebraic and differential/difference equations of the form (1), numerical solution of the model matching problem e.t.c. ([A10], [A12], [A13], [A24], [A25], [A26], [A35], [A36], [B19], [B24], [B27], [B29], [B36], [B39], [B41], [B46], [B49], [B51]).
- c) the division of polynomial matrices ([A21], [B25]).
- d) the determination of the finite (resp. infinite) structure of a rational matrix via a certain row or column reduced matrix fraction description coming from this transfer function matrix. ([A16], [B22]).
- e) the computation of the minimal polynomial of a polynomial matrix ([A33], [B55]), and the greatest common divisor of bivariate polynomials ([A38], [B60]).
- f) the Newton polynomial interpolation of bivariate functions with applications to control theory ([A46], [A49], [B66], [B69]).

3. Computer Aided Control Systems Design

Development of a Computer Aided Design Suite for Modeling, Analysis, Synthesis and Design of Discrete time Automatic Feedback Control Systems based on Symbolic Computations Software Technology. The symbolic processing software that were used are *Maple* and *Mathematica*. A huge number of subroutines have been created in the above computer algebra systems that are based on existing research as well as on the results based on the research of the authors and his colleagues which has already mentioned in Sections 1 and 2. All these programs have been created by the Control group of the Department of Mathematics of the Aristotle University of Thessaloniki and has been funded by the Greek Secretariat of Research and Technology and European Projects. The ultimate goal of all these program packages is their use in education and industry ([A12], [A25], [A34], [A42], [B20], [B21], [B23], [B27], [B29], [B33], [B38], [B40], [B48], [B62], [B63], [B64]). The latest program package was created in cooperation with Wolfram Research Inc. which is the company that built the symbolic software package Mathematica. The packages created during the last years involve the following subjects :

- Manipulation and solution of polynomial and rational matrix equations i.e. solution of rational matrix Diophantine equations over several rings (ring of polynomials, proper rational functions, proper and stable rational functions e.t.c). We note that solutions of Diophantine equations under those rings provide solutions to important problems such as the design of stabilizing compensators, decoupling, model matching etc.
- Linear model descriptions i.e. representations of descriptor systems and tools for transformations between other types of models, such as generalized state space systems, polynomial matrix descriptions, e.t.c.
- System analysis i.e. computations of various types of invariants such as decoupling zeros, system zeros and poles, system properties e.g. controllability, reachability, observability, Smith-McMillan forms over different rings e.t.c.
- Time and frequency domain characteristics, i.e. state and output time responses to various types of inputs, nyquist plots, etc.
- Synthesis and design techniques i.e. stabilizing compensators, model matching, decoupling, asymptotic tracking, pole assignment, e.t.c.

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